DFI2 Stabilization & Scanner Requirements

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INTRODUCTION

This paper presents a method and derivation for determining motor, encoder, and servo parameters for an optical stabilization and scanning system. In addition to generalized solutions, specific parameters for HSI and LFI are calculated.

HSI is a passive HyperSpectral Imager (same as LASH) and LFI is an active Laser Fluorescence Imager. These two components make up the DFI2 system.

BACKGROUND

Data taken from a Piper Aztec flight [Even, et. al.] showed that 99% of the time rotational velocities were well below the 10 deg/s goal. They were:

Parameter	Value	Units
ω_{roll}	4	deg/s
ω_{pitch}	3	deg/s
ω_{yaw}	9	deg/s

Interestingly, yaw had the greatest velocities. The mirror servo system must take this into account.

HSI/LASH

The new stabilization system for LASH will be used for HSI (and most likely other future systems as well). The requirements for LASH are the strictest and used for this analysis. They are:

Parameter	Value	Units
Field of View	+/-20	deg
Aircraft Roll Range	+/-15	deg
Aircraft Roll Rate	10	deg/s
Point-to-Track Range	+/-30	deg
Aircraft Pitch Range	+/-15	deg
Aircraft Pitch Rate	10	deg/s
PitchAhead/Back Range	+/-20	deg
Aircraft Yaw Range	+/-15	deg
Aircraft Yaw Rate	10	deg/s

Notice if you combine stabilization and point-to-track functions the roll angle can get up to +/-45 degrees. The window opening in the pod must be quite large. Also, the outer edge of the pushbroom can be up to +/-50 degrees! Personally I think most of these numbers are too large and therefore have reduced the stabilization requirements for the LFI scanner (next section).

Stabilization Criteria

The original goal for pointing accuracy is $1/10^{th}$ pixel. To maintain such accuracy during flight is overly difficult and unnecessary (as will be shown later). A reasonable criterion is that the captured image of a straight object such as a road remains straight. A simple hypothesis states that as long as the image falls within the bound of a CCD pixel over time then the resulting picture will appear stable. This implies that the center of an object's image must not shift by more than ½ pixel. Note that this error is not on a frame by frame basis but valid for an infinite number of frames. Since a pixel is smeared (elongated) in the direction of flight we usually bin spatially by 2:1 in order to get relatively "square" pixels in the captured picture. If an unbinned pixel is roughly 1 mrad in size then our ½ binned pixel stabilization criteria works out to be +/-1 mrad.

Spot Size

An optical analysis can show the effects of ½ pixel error. The on/off pixel modulation (unbinned) from the spectrometer is about 25% [Anderson]. This means that there is some spreading of light – the spectrometer has an equivalent spot size. Modulation is defined by

$$M := \frac{L_{max} - L_{min}}{L_{max} + L_{min}}$$

If we assume a linear system then $L_{max} + L_{min} = 1$. Solving we get $L_{max} = 62.5\%$ and $L_{min} = 37.5\%$. The modulation is a convolution of the spectrometer equivalent spot and the CCD pixels. Figure 1 shows the results of numerical convolutions for gaussian spot sizes (FWHM) of 1.0, 1.2, and 1.4 pixels. Obviously the greater the spot size the lesser the modulation.

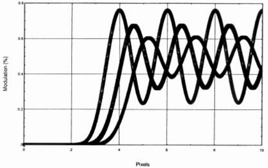

Figure 1. Convolution of spectrometer spot and CCD pixels.

At a FWHM of about 1.4 pixels we get 25% modulation. The standard deviation σ of a unity amplitude gaussian is determined by

$$\sigma := \sqrt{\frac{-\left(FWHM^2\right)}{4 \cdot \ln(0.5)}}$$

which gives a σ of about 0.84 pixels. The resulting spot profile relative to CCD pixel size is shown in Figure 2. It can be seen that even the smallest features (such as lines painted on a road) have their luminous energy spread out across several pixels. Spatial resolution is limited mainly by the spectrometer, not the CCD.

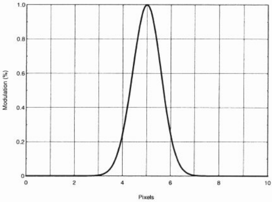

Figure 2. Spectrometer spot size in CCD pixel units.

The amplitude of each pixel is equal to the area under the spot profile curve above it. Figure 3 compares pixel amplitudes for a spot centered on a pixel versus a spot centered between pixels – a change of ½ pixel.

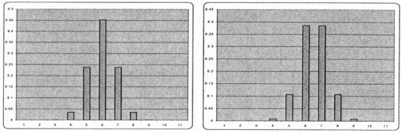

Figure 3. Comparison of pixel amplitudes versus spot position.

The energy is spread across 4 unbinned pixels. A change of 1 pixel (that's 1 mrad of stabilization error) will be relatively undetectable. In fact, an error of 2 mrad is not unreasonable. Therefore, a specification for stabilization error of +/-1 mrad is chosen. To maintain stability the measurement of position should have a resolution of at least 4 times greater than this. Finally, the stabilization goals are:

Parameter	Value	Units
Stabilization Tracking Error	1	mrad
Stabilization Resolution	0.25	mrad

Aircraft Motion

Analysis of stabilization is easier if done in terms of sinewaves. Given an amplitude (maximum angle θ_{plane}) and a rate (rotational velocity ω_{plane}) the maximum undistorted sinewave has a frequency of

$$f_{plane} := \frac{\omega_{plane}}{2 \cdot \pi \cdot \theta_{plane}}$$

Using $\theta_{plane} = 0.261$ rad (15 deg)and $\omega_{plane} = 0.175$ rad/s (10 deg/s) we get an aircraft frequency of about 0.1 Hz. This is a rather slow sinewave but is nonetheless representative of actual flight. Position can be defined by

$$\theta(t) := \theta_{plane} \cdot \sin\left(2 \cdot \pi \cdot f_{plane} \cdot t\right)$$

Substituting, we get

$$\theta(t) := \theta_{plane} \cdot \sin\left(\frac{\omega_{plane} \cdot t}{\theta_{plane}}\right)$$

This is a sinewave function of maximum undistorted amplitude. Maximum acceleration occurs at the peaks and is zero as position passes through nadir. Acceleration is determined by differentiating twice

$$\alpha(t) := -\left(\frac{\omega_{plane}^{2}}{\theta_{plane}}\right) \cdot \sin\left(\frac{\omega_{plane} \cdot t}{\theta_{plane}}\right)$$

Ignoring the sine term and polarity the maximum acceleration is simply

$$\alpha_{plane} := \frac{\omega_{plane}^{2}}{\theta_{plane}}$$

or about 0.12 rad/s^2. Note that when applying these equations to mirrors the parameters may be ½ as great if there is a 2x optical magnification. For roll there is no optical magnification using a gimbal mounted mirror.

Stabilization Tracking Error

One drawback of dynamic stabilization is that there will always be some finite tracking error. By the time the servo loop has moved the mirrors to the correct position it is already too late, the aircraft has moved on. It is like trying to shoot a duck. If you aim at the duck you never hit it. It is this inherent time delay that causes tracking error. The servo loop can easily keep up with the very slow 0.175 rad/s (10 deg/s) rate of change but it does so a fraction in time later than needed. The maximum rate of change is $\Delta\theta/\Delta t$ or

$$\omega_{plane} := \frac{\theta_{error}}{t_{delay}}$$

Rearranged we get

$$t_{delay} := \frac{\theta_{error}}{\omega_{plane}}$$

The allowable servo delay is then 1 mrad divided by 0.175 rad/s or 6 ms. The equivalent –3dB full power bandwidth of the servo loop can be determined from time delay by

$$f_{3dB} := \frac{1}{\pi \cdot t_{delay}}$$

The servo loop must operate at 27 Hz bandwidth in order to keep the tracking error to less than 1 mrad! This can be confirmed by SPICE simulation (an analog circuit simulator). An input sinewave of 0.1 Hz and 261 mrad amplitude is fed into a low pass filter having a –3dB point of 10 Hz. Figure 4 shows both signals. The difference between the two is virtually impossible to see.

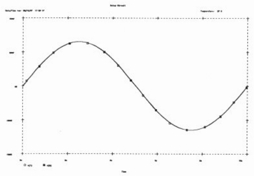

Figure 4. Input and output of servo (SPICE).

However, if we look at the difference between the two signals the error becomes apparent. Figure 5 shows this dynamic error. The peak errors are about 2.5 mrad or 2½ pixels. If the filter is adjusted to 27 Hz then error decreases to +/-1 pixel.

$$f_{mirror} = \frac{\omega_{plane}}{\pi \, \theta_{error}}$$

$$t_r = 2.2 \, \frac{\theta_{error}}{\omega_{plane}}$$

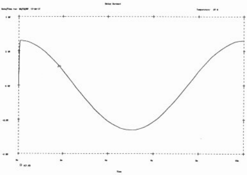

Figure 5. Dynamic position error (SPICE).

Substituting our required –3dB bandwidth formula into the previous equation for position we get

$$\theta(t) := \theta_{plane} \cdot \sin\left(\frac{\omega_{plane}}{\theta_{error}} \cdot t\right)$$

Differentiating twice we get a maximum acceleration of

$$\alpha_{mirror} := \theta_{plane} \cdot \left(\frac{\omega_{plane}}{\theta_{error}}\right)^2$$

which gives us a required acceleration of 8000 rad/s^2 for 1 mrad of tracking error. Ouch! Reducing tracking error to 1/10th pixel increases motor size by a factor of 100. It is helpful to see the relationship between motor size (α) and stabilization error. Figure 6 is a graph of α_{mirror} versus θ_{error}.

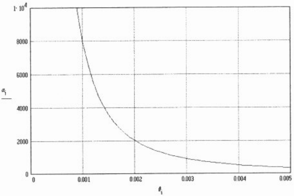

Figure 6. Motor acceleration versus stabilization error.

By changing our error specification from 1 mrad to 2 mrad we reduce motor size and power by a factor of four.

Encoder

To get 0.25 mrad resolution an encoder wheel needs to have thousands of counts per turn. This is determined by multiplying the resolution (counts per radian) by 2π or

$$n_{encoder} := \frac{2 \cdot \pi}{0.25 \cdot 10^{-3}}$$

which gives 25k counts per revolution. Higher counts than this are useless and only increase cost and complexity. For both pitch and yaw which have 2x optical magnification of angle the encoder counts must be doubled.

One problem with encoder wheels is that they are not usually hollow. Both the roll and yaw axis' require an encoder wheel with a 3 inch hole in the middle of it. Accurately mounting such a wheel could prove to be quite difficult.

Miscellaneous

Topics rarely covered but extremely important is how to handle power up initialization and limits. There must be hard limits to prevent the mirrors from exceeding their desired angular ranges. Soft limits help prevent damage to the mirror by indicating to the servo or software that the valid range has been violated before a hard limit is reached.

During a power up sequence the servo does not know the exact position of the differential encoder and therefore the mirror. An index can be used to reset position to a known reference but it must be found before damage occurs. Power up transients may cause the mirror to slam hard against a stop. Proper sequencing of software and servo power supplies must be done correctly to prevent damage.

LFI

The LFI scanner problem is more difficult for two reasons: 1) Mirror size and moment is much greater, 2) Scanning motion is added to stabilization. Fortunately, our "pixel" spacing is about 20 times greater than the HSI system so the ½ pixel stabilization error criteria works out to be +/-10 mrad.

The scanner consists of a single mirror mounted on a two axis gimbal for pitch and roll. It provides both raster scanning and stabilization to create a geo-referenced square data grid. LFI mission specifications are:

Parameter	Value	Units
Altitude	800	m
Airspeed	45	m/s
Laser Rate	60	Hz
Scan Field of Regard	+/-20	deg
Point-to-Track Range	+/-10	deg
Pitch Ahead/Back Range	+0, -20	deg

In order to keep mirror size to a minimum I have reduced the stabilization and pointing specifications to the following:

Parameter	Value	Units
Aircraft Roll Range	+/-10	deg
Aircraft Roll Rate	10	deg/s
Aircraft Pitch Range	+/-5	deg
Aircraft Pitch Rate	5	deg/s
Aircraft Yaw Range	+/-10	deg
Aircraft Yaw Rate	10	deg/s

Scanning

Scanning is performed by rolling the mirror side-to-side in sync with the laser shots. An ideal scan pattern is shown in Figure 7.

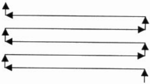

Figure 7. Ideal scan pattern.

In order to keep the lines orthogonal to the direction of flight the pitch must sweep backwards during scan to compensate for the aircraft's forward motion. At the end of each line the pitch must return forward prior to the start of the next line. There are, of course, no sharp corners in the scan motion. The resulting mirror path looks like a Figure 8 with the center crossover at nadir.

Figure 8. Actual mirror path.

The laser firing rate must be held constant to optimize laser performance so the line scan velocity must be constant (it's actually an arctangent function but the error here is negligible). Combining the scan path and laser shots we get a ground referenced square grid pixel pattern as shown in Figure 9. Due to finite retrace times one or more laser shots will be wasted at the end of each line.

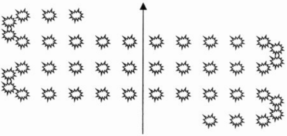

Figure 9. Actual grid pattern of laser shots.

Square Grid

To achieve a square grid pattern the spacing of shots along the scan must be equal to the spacing in the direction of flight. The forward spacing is simply

$$\Delta x := v_{plane} \cdot \left(T_{line} + T_{retrace}\right)$$

Let n be the number of shots per line and m the number of shots during retrace. There are then (n-1) segments per line and (m+1) segments during retrace. Line spacing depends on altitude A_{plane} and is given by

$$\Delta x := \frac{2 \cdot A_{plane}}{(n-1)} \cdot \tan\left(\frac{\theta_{line}}{2}\right)$$

where θ_{line} is 0.698 rad (40 deg). Laser frequency is

$$f_{laser} := \frac{n+m}{T_{line} + T_{retrace}}$$

Combining all three equations we get

$$\frac{v_{plane} \cdot (n+m)}{f_{laser}} = \frac{2 \cdot A_{plane}}{(n-1)} \cdot \tan\left(\frac{\theta_{line}}{2}\right)$$

Finally, solving for n

$$n := \frac{1 - m + \sqrt{(m+1)^2 + \frac{8 \cdot A_{plane} \cdot f_{laser}}{v_{plane}} \cdot \tan\left(\frac{\theta_{line}}{2}\right)}}{2}$$

So, for example, if 4 shots are wasted during retrace we get 26 shots per line (must round to nearest integer). This results in a rectangular grid spacing of about 23 m.

The roll scan frequency of the mirror is given by

$$f_{scan} := \frac{f_{laser}}{2\cdot(n+m)}$$

which is 1 Hz. This should be low enough such that the gimbal assembly will not shake itself apart.

Roll Retrace

The scan line retrace (roll axis) can be modeled as a ½ sinewave function. Retrace must go from $+\omega_{line}$ to $-\omega_{line}$ in one retrace time as shown in Figure 10.

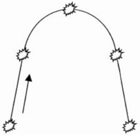

Figure 10. Retrace function.

The retrace time is

$$T_{retrace} := \frac{m+1}{f_{laser}}$$

Applying this to the formula for a ½ sinewave we get

$$\theta(t) := \theta_{retrace}\cdot\sin\left(\frac{f_{laser}\cdot\pi}{m+1}\cdot t\right) + k_{offset}$$

where k_{offset} is at 20 degrees. The line scan velocity ω_{line} is

$$\omega_{line} := \frac{\theta_{line}\cdot f_{laser}}{n-1}$$

which is equal to $\delta\theta(t)/\delta t$ at $t = 0$. Solving for $\theta_{retrace}$

$$\theta_{retrace} := \frac{\theta_{line}\cdot(m+1)}{\pi\cdot(n-1)}$$

Finally, the formula for a sinewave retrace is

$$\theta(t) := \frac{\theta_{line}\cdot(m+1)}{\pi\cdot(n-1)}\cdot\sin\left(\frac{f_{laser}\cdot\pi}{m+1}\cdot t\right) + k_{offset}$$

Differentiating twice the peak acceleration is found to be

$$\alpha_{roll} := \frac{\theta_{line}\cdot f_{laser}^{2}\cdot\pi}{(n-1)\cdot(m+1)}$$

Nonlinear Roll Retrace

In our example with $n = 26$, $m = 4$, $f_{laser} = 60$, and $\theta_{line} = 0.698$ rad, the peak roll retrace acceleration is about 63 rad/s^2. This works out great for a linear servo system because the acceleration is not required to change instantaneously but rather as a sine function itself. There is, however, a servo trick [Davies] we can play to maintain performance while reducing motor size. And that is to go open loop during retrace by driving the motor to full torque with a separate control signal. As soon as retrace is done control is returned to the loop. During retrace the feedback signal to the servo must not be integrated or there will be a stored error signal which causes a ring in the settling response of the next line. A nice trick but it requires some very careful tuning of the servo system.

At full motor acceleration the scan path followed during retrace is a parabola. The scan must decelerate from ω_{line} to zero in ½ retrace time. Since $\alpha = \delta\omega/\delta t$ this gives

$$\alpha_{roll} := \frac{\theta_{line}\cdot f_{laser}^{2}\cdot 2}{(n-1)\cdot(m+1)}$$

This is a 36% reduction in motor size for the same retrace time. Our servo motor is now 40 rad/s^2.

Nonlinear Pitch Retrace

Pitch retrace must move ½ grid space $\Delta\theta$ in ½ retrace time. It accelerates from essentially zero velocity to some large velocity and then decelerate back to zero in one retrace time. Position is defined by

$$\Delta\theta := \frac{1}{2}\cdot\alpha_{pitch}\cdot T_{retrace}^{2}$$

Because of a 2x optical magnification for pitch the angle traveled for one grid is cut in half. It is then given as

$$\theta_{grid} := \operatorname{atan}\left(\frac{\Delta x}{2\cdot A_{plane}}\right)$$

Combining with the equation for retrace time and solving for α_{pitch} we get

$$\alpha_{pitch} := \frac{4\cdot f_{laser}^{2}}{(m+1)^{2}}\cdot\operatorname{atan}\left(\frac{\Delta x}{2\cdot A_{plane}}\right)$$

which results in a required acceleration of 8 rad/s^2. This is for an open loop solution. To keep the servo linear (which may be way easier) acceleration goes up by a factor of $\pi/2$ to obtain 13 rad/s^2.

Both pitch and roll accelerations are functions of m. Obviously, greater retrace time allows lower acceleration and motor size, however, the number of points per line goes down. A compromise must be reached. Both functions are plotted in Figure 11. Roll is the worst case with motor size increasing greatly at less than 3 wasted shots at end of line.

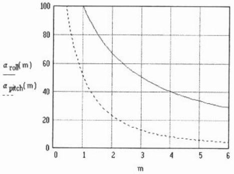

Figure 11. Accelerations versus m (wasted shots).

Stabilization

Using the required accelerations for pitch and roll retracing the stabilization tracking error can be determined. Rearranging our stabilization formula

$$\alpha_{mirror} := \theta_{plane} \cdot \left(\frac{\omega_{plane}}{\theta_{error}} \right)^2$$

and solving for θ_{error} we get

$$\theta_{error} := \omega_{plane} \cdot \sqrt{\frac{\theta_{plane}}{\alpha}}$$

Applying a roll of 0.175 rad (10 deg), a rate of 0.175 rad/s, and an acceleration of 40 rad/s^2 we get an error of 12 mrad. This is about ½ grid spacing -- roughly what we need. For pitch since there is 2x optical magnification so at 8 rad/s^2 we get 9 mrad error (using 10 degree yaw range). Fortunately both of these results are reasonable which means the motor sizes do not need to be increased just because of stabilization. Basically, if it can scan, then stabilization is free.

Yaw correction is performed almost entirely with pitch. This is needed to keep the scan lines orthogonal to the direction of flight.

SUMMARY

The following tables summarize the motor, encoder, and servo requirements for DFI2 passive and active stabilization/scanning systems. They are based on 2 mrad tracking error for HSI and scanner retrace for LFI. Both systems use linear servos.

HSI

Parameter	Value	Units
Roll Acceleration	2,000	rad/s^2
Roll Encoder Resolution	25,000	counts/rev
Roll Servo Bandwidth	14	Hz
Pitch Acceleration	1,000	rad/s^2
Pitch Encoder Resolution	50,000	counts/rev
Pitch Servo Bandwidth	14	Hz
Yaw Accelerations	1,000	rad/s^2
Yaw Encoder Resolution	50,000	counts/rev
Yaw Servo Bandwidth	14	Hz

LFI

Parameter	Value	Units
Shots per Retrace (m)	3	
Shots per Line (n)	27	
Grid Spacing	22	m
Roll Acceleration	76	rad/s^2
Roll Encoder Resolution	6,000	counts/rev
Roll Servo Bandwidth	2.3	Hz
Pitch Acceleration	19	rad/s^2
Pitch Encoder Resolution	12,000	counts/rev
Pitch Servo Bandwidth	2.3	Hz